Symmetries and Integrability of Difference Equations

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PREFACE

Symmetries and Integrability of Difference Equations

The notion of integrability was first introduced in the 19th century in the context of classical mechanics with the definition of Liouville integrability for Hamiltonian flows. Since then, several notions of integrability have been introduced for partial and ordinary differential equations. Closely related to integrability theory is the symmetry analysis of nonlinear evolution equations. Symmetry analysis takes advantage of the Lie group structure of a given equation to study its properties. Together, integrability theory and symmetry analysis provide the main method by which nonlinear evolution equations can be solved explicitly.

Difference equations (DE), like differential equations, are important in numerous fields of science and have a wide variety of applications in such areas as mathematical physics, computer visualization, numerical analysis, mathematical biology, economics, combinatorics, and quantum field theory. It is thus crucial to develop tools to study and solve DEs. While the theory of symmetry and integrability for differential equations is now largely well-established, this is not yet the case for discrete equations. Although over recent years there has been significant progress in the development of a complete analytic theory of difference equations, further tools are still needed to fully understand, for instance, the symmetries, asymptotics and the singularity structure of difference equations.

The series of SIDE meetings on Symmetries and Integrability of Difference Equations started in 1994. Its goal is to provide a platform for an international and interdisciplinary communication for researchers working in areas associated with integrable discrete systems, such as classical and quantum physics, computer science and numerical analysis, mathematical biology and economics, discrete geometry and combinatorics, theory of special functions, etc. The previous SIDE meetings took place in Estérel near Montréal, Canada (1994), at the University of Kent in Canterbury, UK (1996), in Sabaudia near Rome, Italy (1998), at the University of Tokyo, Japan (2000), in Giens, France (2002), and in Helsinki, Finland (2004). The SIDE VII meeting was held at the University of Melbourne from 10–14 July 2006. The scientific committee consisted of Nalini Joshi (The University of Sydney), Frank W Nijhoff (University of Leeds), Reinout Quispel (La Trobe University) and Colin Rogers (University of New South Wales). The local organization was in the hands of John A G Roberts and Wolfgang K Schief.

Proceedings of all the previous SIDE meetings have been published; the 1994 and 1988 meetings (edited respectively by D Levi, L Vinet and P Winternitz, and by D Levi and O Ragnisco) as volumes of the CRM Proceedings and Lecture Notes (AMS Publications), the 1996 meeting (edited by P Clarkson and F W Nijhoff) as Volume 255 in the LMS Lecture Note Series. Starting from the 1996 meeting the formula of publication has been changed to include rather selected refereed contributions submitted in response to a call for papers issued after the meetings and not restricted to their participants. Thus publications reflecting the scope of the 1996 meeting (edited by J Hietarinta, F W Nijhoff and J Satsuma) appeared in *Journal of Physics A: Mathematical and General* **34** 48 (special issue), and of the 1998 and 2000 meetings (edited respectively by F W Nijhoff, Yu B Suris and C-M Viallet, and by J F van Diejen and R Halburd) in *Journal of Nonlinear Mathematical Physics* **10** (Suppl. 2) and **12** (Suppl. 2).

The aim of this special issue is to benefit from the occasion offered by the SIDE VII meeting, producing an issue containing papers which represent the state-of-the-art knowledge

for studying integrability and symmetry properties of difference equations. This special issue features high quality research papers and invited reviews which deal with themes that were covered by the SIDE VII conference. These are in alphabetical order:

Algebraic-geometric approaches to integrability. The first section contains a paper by T Hamamoto and K Kajiwara on hypergeometric solutions to the q-Painlevé equation of type $A_4^{(1)}$.

Discrete geometry. In this category there are three papers. J Cielinski offers a geometric definition and a spectral approach on pseudospherical surfaces on time scales, while A Doliwa considers generalized isothermic lattices. The paper by U Pinkall, B Springborn and S Weißmann is concerned with a new doubly discrete analogue of smoke ring flow and the real time simulation of fluid flow.

Integrable systems in statistical physics. Under this heading there is a paper by R J Baxter on corner transfer matrices in statistical mechanics, and a paper by S Boukraa, S Hassani, J-M Maillard, B M McCoy, J-A Weil and N Zenine where the authors consider Fuchs-Painlevé elliptic representation of the Painlevé VI equation.

KP lattices and differential-difference hierarchies. In this section we have seven articles. C R Gilson, J J C Nimmo and Y Ohta consider quasideterminant solutions of a non-Abelian Hirota-Miwa equation, while B Grammaticos, A Ramani, V Papageorgiou, J Satsuma and R Willox discuss the construction of lump-like solutions of the Hirota-Miwa equation. J Hietarinta and C Viallet analyze the factorization process for lattice maps searching for integrable cases, the paper by X-B Hu and G-F Yu is concerned with integrable discretizations of the (2+1)-dimensional sinh-Gordon equation, and K Kajiwara, M Mazzocco and Y Ohta consider the Hankel determinant formula of the τ-functions of the Toda equation. Finally, V G Papageorgiou and A G Tongas study Yang-Baxter maps and multi-field integrable lattice equations, and H-Y Wang, X-B Hu and H-W Tam consider the two-dimensional Leznov lattice equation with self-consistent sources.

Quantum integrable systems. This category contains a paper on q-extended eigenvectors of the integral and finite Fourier transforms by N M Atakishiyev, J P Rueda and K B Wolf, and an article by S M Sergeev on quantization of three-wave equations.

Random matrix theory. This section contains a paper by A V Kitaev on the boundary conditions for scaled random matrix ensembles in the bulk of the spectrum.

Symmetries and conservation laws. In this section we have five articles. H Gegen, X-B Hu, D Levi and S Tsujimoto consider a difference-analogue of Davey-Stewartson system giving its discrete Gram-type determinant solution and Lax pair. The paper by D Levi, M Petrera, and C Scimiterna is about the lattice Schwarzian KDV equation and its symmetries, while O G Rasin and P E Hydon study the conservation laws for integrable difference equations. S Saito and N Saitoh discuss recurrence equations associated with invariant varieties of periodic points, and P H van der Kamp presents closed-form expressions for integrals of MKDV and sine-Gordon maps.

Ultra-discrete systems. This final category contains an article by C Ormerod on connection matrices for ultradiscrete linear problems.

We would like to express our sincerest thanks to all contributors, and to everyone involved in compiling this special issue.

Adam Doliwa, Risto Korhonen and Stéphane Lafortune
Guest Editors